12-7 The Normal Distribution

Main Ideas

- Determine whether a set of data appears to be normally distributed or skewed.
- Solve problems involving normally distributed data.

New Vocabulary

discrete probability distribution
continuous probability distribution
normal distribution
skewed distribution

GET READY for the Lesson

The frequency table below lists the heights of the 2005 New England Patriots. However, it does not show how these heights compare to the average height of a professional football player. To make that comparison, you can determine how the heights are distributed.

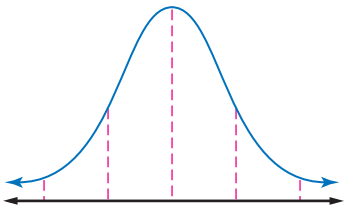
Height (in.)	70	71	72	73	74	75	76	77	78	79	80
Frequency	13	3	5	7	10	9	14	2	4	0	1

Source: www.nfl.com

Normal and Skewed Distributions The probability distributions you have studied thus far are **discrete probability distributions** because they have only a finite number of possible values. A discrete probability distribution can be represented by a histogram. For a **continuous probability distribution**, the outcome can be any value in an interval of real numbers. Continuous probability distributions are represented by curves instead of histograms.

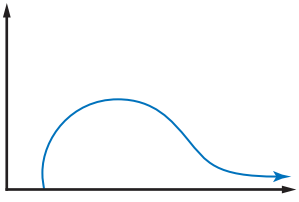
The curve at the right represents a continuous probability distribution. Notice that the curve is symmetric. Such a curve is often called a *bell curve*. Many distributions with symmetric curves or histograms are **normal distributions**.

Normal Distribution

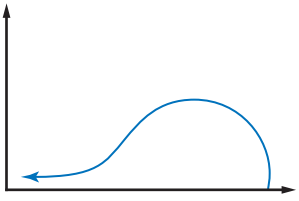


A curve or histogram that is not symmetric represents a **skewed distribution**. For example, the distribution for a curve that is high at the left and has a tail to the right is said to be *positively skewed*. Similarly, the distribution for a curve that is high at the right and has a tail to the left is said to be *negatively skewed*.

Positively Skewed



Negatively Skewed



Study Tip

Skewed Distributions

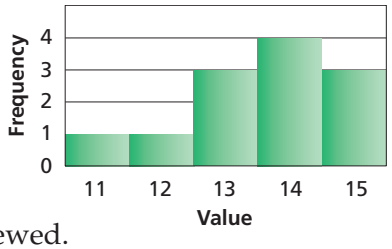
In a positively skewed distribution, the long tail is in the positive direction. These are sometimes said to be *skewed to the right*. In a negatively skewed distribution, the long tail is in the negative direction. These are sometimes said to be *skewed to the left*.

EXAMPLE Classify a Data Distribution

1 **Determine whether the data {14, 15, 11, 13, 13, 14, 15, 14, 12, 13, 14, 15} appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.**

Make a frequency table for the data. Then use the table to make a histogram.

Value	11	12	13	14	15
Frequency	1	1	3	4	3



Since the histogram is high at the right and has a tail to the left, the data are negatively skewed.

Concepts in Motion
Animation
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CHECK Your Progress

1. Determine whether the data {25, 27, 20, 22, 28, 20, 24, 22, 20, 21, 21, 26} appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

Use Normal Distributions Standardized test scores, the lengths of newborn babies, the useful life and size of manufactured items, and production levels can all be represented by normal distributions. In all of these cases, the number of data values must be large for the distribution to be approximately normal.

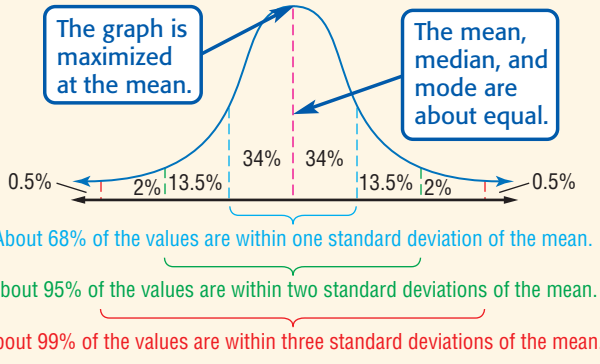
Study Tip

Normal Distributions

If you randomly select an item from data that are normally distributed, the probability that the one you pick will be within one standard deviation of the mean is 0.68. If you do this 1000 times, about 680 of those picked will be within one standard deviation of the mean.

KEY CONCEPT Normal Distribution

Normal distributions have these properties.

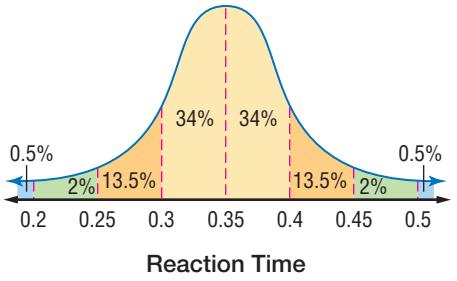


EXAMPLE Normal Distribution

2 **PHYSIOLOGY** **The reaction times for a hand-eye coordination test administered to 1800 teenagers are normally distributed with a mean of 0.35 second and a standard deviation of 0.05 second.**

a. About how many teens had reaction times between 0.25 and 0.45 second?

Draw a normal curve. Label the mean and the mean plus or minus multiples of the standard deviation.



(continued on the next page)

Reading Math

Normally Distributed Random Variable

A *normally distributed random variable* is a variable whose values are arbitrary but whose statistical distribution is normal.

The values 0.25 and 0.45 are 2 standard deviations *below and above* the mean, respectively. Therefore, about 95% of the data are between 0.25 and 0.45. Since $1800 \times 95\% = 1710$, we know that about 1710 of the teenagers had reaction times between 0.25 and 0.45 second.

b. What is the probability that a teenager selected at random had a reaction time greater than 0.4 second?

The value 0.4 is one standard deviation above the mean. You know that about 100% − 68% or 32% of the data are more than one standard deviation away from the mean. By the symmetry of the normal curve, half of 32%, or 16%, of the data are more than one standard deviation above the mean.

The probability that a teenager selected at random had a reaction time greater than 0.4 second is about 16% or 0.16.

CHECK Your Progress

In a recent year, the mean and standard deviation for scores on the ACT were 21.0 and 4.7. Assume that the scores were normally distributed.

2A. If 1,000,000 people took the test, about how many of them scored between 16.3 and 25.7?

2B. What is the probability that a test taker scored higher than 30.4?

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CHECK Your Understanding

Example 1 (p. 725)

1. The table at the right shows recent composite ACT scores. Determine whether the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

Score	Percent of Students
33–36	1
28–32	9
24–27	19
20–23	29
16–19	27
13–15	12

Source: ACT.org

Example 2 (pp. 725–726)

For Exercises 2–4, use the following information.

Mr. Bash gave a quiz in his social studies class. The scores were normally distributed with a mean of 21 and a standard deviation of 2.

2. What percent would you expect to score between 19 and 23?

3. What percent would you expect to score between 23 and 25?

4. What is the probability that a student chosen at random scored between 17 and 25?

QUALITY CONTROL **For Exercises 5–8, use the following information.**

The useful life of a certain car battery is normally distributed with a mean of 100,000 miles and a standard deviation of 10,000 miles. The company makes 20,000 batteries a month.

5. About how many batteries will last between 90,000 and 110,000 miles?

6. About how many batteries will last more than 120,000 miles?

7. About how many batteries will last less than 90,000 miles?

8. What is the probability that if you buy a car battery at random, it will last between 80,000 and 110,000 miles?

Exercises

HOMEWORK HELP	
For Exercises	See Examples
9–11	1
12–23	2

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

9.

U.S. Population	
Age	Percent
0–19	28.7
20–39	29.3
40–59	25.5
60–79	13.3
80–99	3.2
100+	0.0

Source: U.S. Census Bureau

10.

Record High U.S. Temperatures	
Temperature (°F)	Number of States
100–104	3
105–109	8
110–114	16
115–119	13
120–124	7
125–129	2
130–134	1

Source: *The World Almanac*

11. SCHOOL The frequency table at the right shows the grade-point averages (GPAs) of the juniors at Stanhope High School. Do the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain.

GPA	Frequency
0.0–0.4	4
0.5–0.9	4
1.0–1.4	2
1.5–1.9	32
2.0–2.4	96
2.5–2.9	91
3.0–3.4	110
3.5–4.0	75

HEALTH For Exercises 12 and 13, use the following information.
The cholesterol level for adult males of a specific racial group is normally distributed with a mean of 4.8 and a standard deviation of 0.6.

12. About what percent of the males have cholesterol below 4.2?

13. About how many of the 900 men in a study have cholesterol between 4.2 and 6.0?

Real-World Link
Doctors recommend that people maintain a total blood cholesterol of 200 mg/dL or less.

Source: americanheart.org

VENDING For Exercises 14–16, use the following information.
A vending machine usually dispenses about 8 ounces of coffee. Lately, the amount varies and is normally distributed with a standard deviation of 0.3 ounce.

14. What percent of the time will you get more than 8 ounces of coffee?

15. What percent of the time will you get less than 8 ounces of coffee?

16. What percent of the time will you get between 7.4 and 8.6 ounces of coffee?

MANUFACTURING For Exercises 17–19, use the following information.
The sizes of CDs made by a company are normally distributed with a standard deviation of 1 millimeter. The CDs are supposed to be 120 millimeters in diameter, and they are made for drives 122 millimeters wide.

17. What percent of the CDs would you expect to be greater than 120 millimeters?

18. If the company manufactures 1000 CDs per hour, how many of the CDs made in one hour would you expect to be between 119 and 122 millimeters?

19. About how many CDs per hour will be too large to fit in the drives?

FOOD For Exercises 20–23, use the following information.
The shelf life of a particular snack chip is normally distributed with a mean of 180 days and a standard deviation of 30 days.

20. About what percent of the products last between 150 and 210 days?

21. About what percent of the products last between 180 and 210 days?

22. About what percent of the products last less than 90 days?

23. About what percent of the products last more than 210 days?

EXTRA PRACTICE
See pages 918, 937.

Math Online
Self-Check Quiz at algebra2.com

RAINFALL For Exercises 24–26, use the table at the right.

24. Find the mean.

25. Find the standard deviation.

26. If the data are normally distributed, what percent of the time will the annual precipitation in these cities be between 16.97 and 7.69 inches?

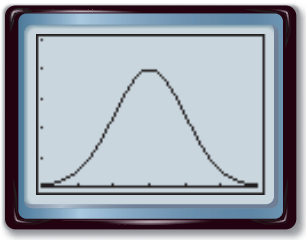
Average Annual Precipitation	
City	Precipitation (in.)
Albuquerque	9
Boise	12
Phoenix	8
Reno	7
Salt Lake City	17
San Francisco	20

Source: noaa.gov

H.O.T. Problems

27. OPEN ENDED Sketch a positively skewed graph. Describe a situation in which you would expect data to be distributed this way.

28. CHALLENGE The graphing calculator screen shows the graph of a normal distribution for a large set of test scores whose mean is 500 and whose standard deviation is 100. If every test score in the data set were increased by 25 points, describe how the mean, standard deviation, and graph of the data would change.



[200, 800] scl: 100 by [0, 0.005] scl: 0.001

29. *Writing in Math* Use the information on page 724 to explain how the heights of professional athletes are distributed. Include a histogram of the given data, and an explanation of whether you think the data is normally distributed.

STANDARDIZED TEST PRACTICE

30. ACT/SAT If $x + y = 5$ and $xy = 6$, what is the value of $x^2 + y^2$?

A 13

B 17

C 25

D 37

31. REVIEW Jessica wants to create several different 7-character passwords. She wants to use arrangements of the first three letters of her name, *followed* by arrangements of 4 digits in 1987, the year of her birth. How many different passwords can she create?

F 672 **G** 288 **H** 576 **J** 144

Spiral Review

Find the variance and standard deviation of each set of data to the nearest tenth. (Lesson 12-6)

32. {7, 16, 9, 4, 12, 3, 9, 4}

33. {12, 14, 28, 19, 11, 7, 10}

A card is drawn from a standard deck of cards. Find each probability. (Lesson 12-5)

34. P(jack or queen)

35. P(ace or heart)

36. P(2 or face card)

GET READY for the Next Lesson

PREREQUISITE SKILL **Use a calculator to evaluate each expression to four decimal places.** (Lesson 9-5)

37. e^{-4}

38. e^{3}

39. $e^{\frac{1}{2}}$